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# Examining the Commonality in Liquidity and Volatility Risk

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**Abstract** We estimate latent factor models of liquidity and volatility. Common liquidity and volatility factors are extracted using multiple liquidity and volatility measures. Additionally, a latent factor is extracted by aggregating across both liquidity and volatility resulting in what we call the common uncertainty factor. The underlying uncertainty factor is correlated with the individual and the common liquidity and volatility factors as well as returns. We find that the underlying uncertainty risk factor is significantly priced in the cross section of expected returns, while the risks associated solely with liquidity and volatility are not suggesting that liquidity risk and volatility risk in stocks are related and should not be considered separately.

**Keywords** Liquidity, Volatility, Uncertainty

**JEL Classification** G1, G12

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# 1. INTRODUCTION

Recently, there have been two separate paths that explore the cross-section of stock returns. One emphasizes the importance of volatility as a systematic risk factor (e.g. Ang et al., 2006; Adrian and Rosenberg, 2006; Moise, 2007), while the other focuses on systematic liquidity risk (see Pástor and Stambaugh, 2003; Acharya and Pedersen, 2005; Chen, 2005; Sadka, 2006). Additionally, since there are many different measures of liquidity, several studies have focused on identifying a common systematic liquidity factor (see Chordia, Roll, and Subrahmanyam, 2000; Hasbrouck and Seppi, 2001; Eckbo and Norli, 2002). While much work has been done focusing on liquidity and volatility separately, relatively little work has been done exploring the joint pricing of systematic liquidity and volatility risk.

Liquidity and volatility arise from differing economic causes, with volatility resulting from fluctuations in asset valuations and liquidity caused by market trading frictions. However, it is possible that they are both proxies for another more fundamental factor, which we will refer to as uncertainty, which varies with the state of the economy. If this is the case, it would be interesting to see if the explanatory power of either liquidity or volatility risk is reduced in a joint asset pricing model. Bandi et al. (2008) examine this question at the market level using measures of market liquidity and volatility risk derived from high frequency prices of the SPDR (a trust invested in the S&P 500). They find that when considering liquidity or volatility risk individually they are significant risk factors, however, in the model which includes both liquidity and volatility risk only the volatility risk is significant. They conclude that this likely results because they each are proxies for a more fundamental underlying factor. This paper will further explore whether a common uncertainty factor derived from liquidity and volatility risk is significantly priced in the cross-section and if this could drive the results when volatility and liquidity risk are considered separately.

To better understand the disparate liquidity measures, Korajczyk and Sadka (2008) examine eight different measures of liquidity to determine whether they are each capturing a common underlying liquidity factor or whether there are potentially multiple liquidity risk factors each captured by a different measure. Using the technique of Connor and Korajczyk (1987), Korajczyk and Sadka (2008) are able to extract an across-measure liquidity factor derived by the stacking of all individual liquidity measures. They find the across-measure liquidity factor is what is significant in the cross section, not anything unique to the various measures.

Using analysis techniques similar to those in Korajczyk and Sadka (2008), we extract latent factors for multiple liquidity and volatility measures on a sample of 4975 NYSE stocks over the period of July 1962 to December 2011. In addition to risk factors specific to each individual measure, across-measure liquidity (volatility) factors are estimated by considering multiple liquidity (volatility) measures. Common, what we will term as "uncertainty," factors are extracted across all of the liquidity and volatility measures. We use these various factors to further examine the joint pricing of liquidity and volatility risk in the cross-section.

One liquidity measure included in the study is the Amihud (2002) measure, the ratio of absolute returns and dollar volume. This is a gauge of price impact since it measures the daily price response for each dollar of trading. Also considered is the relative spread, the ratio of the bid-ask spread and the midpoint price [see Næs, Skjeltorp, and Ødegaard (2011)], the Roll (1984) measure which is based on the autocorrelation of daily returns, and the turnover, the ratio of volume to shares outstanding. The volatility measures are monthly realized variance (sum of squared returns), a monthly measure using the open, close, high, and low prices [see Garman and Klass (1980)], and monthly estimates from a GARCH(1,1) specification.

Our results indicate that there does exist a fundamental uncertainty factor that is related to both systematic liquidity and volatility as well as returns. Pair-wise canonical correlations show that shocks to liquidity and volatility are correlated to the common uncertainty factor and contemporaneously correlated to returns. Liquidity factors are highly persistent while volatility factors exhibit a lower degree of persistence. The shocks to liquidity and volatility factors are estimated as the residuals of an AR(2) model.

The final analysis examines the cross-sectional pricing of liquidity risk, volatility risk, and the common uncertainty risk in addition to the premium on the raw liquidity and volatility levels. The across-measure liquidity and volatility factors are orthogonalized to the common uncertainty factor to better isolate the risk specific to liquidity and volatility. We find that uncertainty risk is significantly priced in the cross-section while the risk attributed solely to liquidity and volatility is not. This suggests that liquidity and volatility risk are both (weak) proxies for an underlying risk factor, we choose to call this uncertainty risk, which drives the significant pricing results when considering liquidity and volatility individually.

The paper is organized as follows. Section 2 discusses the specific liquidity and volatility measures as well as the method for extracting the risk factors. Section 3 presents the AR(2) results and explores both the pair-wise contemporaneous and lead-lag correlations of the risk factors and returns. Section 4 presents the cross-sectional pricing analysis and Section 5 concludes.

## 2. DATA

This paper utilizes data from the daily and monthly CRSP databases for stocks traded on the NYSE between July 1962 to December 2011. Since trading on the NASDAQ uses a different trading mechanism relying heavily on market makers, only stocks traded on the NYSE are considered in the analysis. Additionally, only assets with a CRSP share code of 10 or 11 (ordinary common shares) are considered which will eliminate certificates, Americus Trusts components, ADRs, shares of beneficial interest, closed-end funds, REIT's, and ETFs. Stocks with a price lower than \$1 are excluded as well as those observations with a volume = 0. After the described filtering, we are left with a total of 4975 firms over a total of 594 months.

### 2.1. LIQUIDITY MEASURES

There are a wide range of proposed measures of liquidity. We implement a total of four liquidity measures. The first is the measure based on Amihud

(2002). Define the Amihud measure for stock $i$ in month $t$ as

$$A_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{|r_{i,j}|}{dvol_{i,j}} \tag{1}$$

where $r_{i,j}$ is the return on asset $i$ on day $j$ of month $t$, $d_t$ is the number of trading days in the month, and $dvol_{i,j}$ is the dollar volume for asset $i$ on day $j$ of month $t$. Following both Acharya and Petersen (2005) and Korajczyk and Sadka (2008), the monthly measure $A_{i,t}$ is scaled by the ratio of the market capitalization of the CRSP market index at time $t-1$ and at the reference date of July 1962. In order for the monthly measure to be included in the sample, a stock is required to have at least 15 daily observations.

The second liquidity measure employed is the turnover, the ratio of monthly volume and shares outstanding. It is defined as

$$TO_{i,t} = \frac{\sum_{j=1}^{d_t} vol_{i,j}}{SO_{i,t}} \tag{2}$$

where $SO_{i,t}$ is the number of shares outstanding at the end of month $t$. Once again, it is required that a stock have at least 15 daily observations in month $t$ to be included in the sample.

The relative spread is calculated as the difference between the bid and the ask divided by the midpoint price (average of the bid and ask).

$$RS_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{Ask_{i,j} - Bid_{i,j}}{midpt_{i,j}} \tag{3}$$

This is calculated at the daily frequency and then aggregated by taking the monthly average of the daily measures. The purpose of the relative spread is to measure the implicit cost of trading a small number of shares.

The final liquidity measure employed is that of Roll (1984). Assuming the existence of a constant spread $s$, Roll shows that the spread can be estimated as $\hat{s} = 2\sqrt{-Scov}$ where $Scov$ is the covariance of adjacent daily returns. Roll's liquidity measure is estimated each month using daily returns with a minimum of 15 daily returns required to be included in the estimation. Since this is undefined when $Scov > 0$, those liquidity estimates are set to missing.[1]

## 2.2. VOLATILITY MEASURES

Three different estimates of monthly volatility are employed in the following analysis. The first is an estimate formed from the daily realized variance measure simply defined as

$$RV_{i,t} = \sum_{j=1}^{d_t} r_{i,j}^2 \tag{4}$$

where $r_{i,j}$ is the return of asset $i$ on day $j$ of month $t$ and $d_t$ is the number of trading days in month $t$.

Garman and Klass (1980) find that the best analytic scale-invariant estimator of daily volatility, $\sigma_j^2$, is

$$GK_{i,t} = 0.51(u_j - d_j)^2 - 0.019[c_j(u_j + d_j) - 2u_j d_j] - 0.383c_j^2 \tag{5}$$

where $c_j$ is the closing cost, $u_j$ is the daily high, and $d_j$ is the low. Each of the terms is normalized by subtracting the daily opening price. Once the daily estimates are calculated, the monthly estimate is obtained by summing the daily estimate over the days of the month.

The final estimate of monthly volatility for each asset is obtained by estimating a simple GARCH(1,1) model over an expanding window with a minimum of 24 monthly returns required for estimation. Formally, the monthly variance for our GARCH(1,1) model is defined as

$$r_t = c + \varepsilon_t \qquad \varepsilon \sim N(0, \sigma_t^2) \tag{6a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_i \sigma_{t-1}^2 \tag{6b}$$

[1] Harris (1990) suggests using $\hat{s} = -2\sqrt{Scov}$ when $Scov < 0$, but this would result in a negative spread which would imply a negative transaction cost. Since this isn't meaningful, months with $Scov > 0$ are simply set to missing as in Næs, Skjeltorp, and Ødegaard (2011).

Each estimate of liquidity and volatility is Windsorized at the 1st and 99th cross-sectional percentiles for each month to reduce the effects of outliers[2].

This results in an unbalanced panel of 4 liquidity and 3 volatility measures over 4975 NYSE firms spanning a total of 594 months. The various liquidity (volatility) measures will be used to derive a common liquidity (volatility) factor. A common cross-sectional factor will be extracted from the combined liquidity and volatility measures which we will refer to as the common uncertainty factor.

## 2.3. FACTOR DECOMPOSITION

We will be examining the common uncertainty factor across the various liquidity and volatility measures using a process similar to that of Korajczyk and Sadka (2008). Since the units are not comparable for the various liquidity and volatility measures, each measure is standardized using the mean and standard deviation in the cross section using all available data prior to month $t$. Specifically, let $M^i$ be the $n \times T$ matrix of estimator $i$ (this could be either a liquidity or a volatility estimator). Define $\hat{\mu}^i_{t-1}$ and $\hat{\sigma}^i_{t-1}$ as the cross-sectional mean and standard deviation for measure $i$ estimated for all of the sample up to $t-1$. Then the standardized measure is calculated as $S^i_{j,t} = (M^i_{j,t} - \hat{\mu}^i_{t-1})/\hat{\sigma}^i_{t-1}$. The estimator $S^i$ is assumed to follow the factor model

$$S^i = B^i F^i + \varepsilon^i, \tag{7}$$

where $F^i$ is a $k \times T$ matrix of shocks to the liquidity (volatility) measure that are common across the set of $n$ assets, $B^i$ is a $n \times k$ matrix of sensitivities to the common factor, and $\varepsilon^i$ is the $n \times T$ matrix of asset specific shocks to the liquidity (volatility) measure. Connor and Korajczyk (1986) show that $n$-consistent estimates of the factors, $F^i$, are obtained by calculating the eigenvalues of

$$\Omega^i = \frac{S^{i\prime} S^i}{n}. \tag{8}$$

[2]To illustrate, consider the variance estimate $RV_{i,t}$. Let $RV_t^{99\%}$ be the 99th percentile of all RV estimates for the month $t$. If $RV_{j,t} > RV_t^{99\%}$ then $RV_{j,t}$ is set equal to $RV_t^{99\%}$. Similarly, any monthly measure that is less than $RV_t^{1\%}$ will be set equal to the 1st percentile.

While this estimator relies on a balanced panel, it does vastly simplify the calculations as we are now simply calculating the eigenvectors of a $T \times T$ matrix which is independent of the number of stocks in our sample. To accommodate the fact that our panel is unbalanced, we follow the estimation technique of Connor and Korajczyk (1987) which will essentially estimate the elements of $\Omega$ using only the observed data. All of the missing observations in $S^i$ are replaced with zeros and the resulting balanced panel will be called $S^{i*}$. Define $N^i$ as a $n \times T$ indicator matrix in which each element takes a value of 1 if the element in $S^i$ is observed or 0 if the corresponding element in $S^i$ is missing. Now we can construct an unbalanced equivalent of $\Omega$ that only uses the cross-sectional averages of the observed data.

$$\Omega^{i,u}_{t,\tau} = \frac{(S^{i*\prime} S^{i*})_{t,\tau}}{(N^{i\prime} N^i)_{t,\tau}} \tag{9}$$

The estimates of the $k$ latent factors, $\hat{F}^i$, can be calculated as the eigenvectors ($T \times 1$) of the $k$ largest eigenvalues of $\Omega^{i,u}$. Following Connor and Korajczyk (1986), the eigenvectors are normalized so that the rows have a mean-square of 1.

For each measure, including returns, the first three principal components are extracted using the technique outlined above where the latent factors are calculated as the eigenvalues of equation (8). A time series regression of each stock's liquidity of volatility measures on the three extracted factors can be used to help determine the degree of commonality across different stocks for each measure. The estimated regression is

$$S^i_{j,t} = B^i_{j,\cdot} \hat{F}^i_t + \hat{\varepsilon}^i_{j,t}, \tag{10}$$

where $\hat{F}^i_t$ is the $k$x1 vector of factor estimates for month $t$. Recall, the factors for the liquidity and volatility measures are calculated using the aggregate of all the included securities. The regressions based on equation (10) will help determine if these aggregate factors are significant in explaining the measures for each individual stock. The resulting average $R^2$ values for regressions involving one, two, and three factors for each individual security are reported in Table 1.

These cross-sectional results demonstrate that there is a high degree of commonality within each of the liquidity and volatility measures with the cross-sectional average $R^2$ ranging from 16.1% to 48.6% for one factor and increasing to a range of 35.8% to 69.9% with the inclusion of all three factors. There is little difference between the $R^2$ values for the liquidity and volatility measures. For stock returns, increasing the model from one to three factors results in a much more modest gain in average $R^2$ than for the liquidity and volatility measures.

These results are in line with those of Chordia, Roll, and Subrahmanyam (2000) who show a commonality among quoted and effective spread using data from 1992. They are also consistent with those of Korajczyk and Sadka (2008) who find a similar degree of commonality across various liquidity measures.

In addition to estimating the cross-sectional factors within each measure, common factors across all of the liquidity (volatility) measures are extracted as well. This can be accomplished by stacking the multiple liquidity (volatility) measures and then using the stacked matrix to form $\Omega$. The factors extracted from the stacked liquidity (volatility) measures will be referred to as the common, or across-measure, liquidity (volatility) factors. The sign of the liquidity factors is chosen so that an increase in the factor will correspond to an increase in liquidity. This is done by choosing the sign so that the within-measure factors are negatively correlated with the cross-sectional mean of the measure (although for turnover it will be positively correlated).

To better understand whether liquidity and volatility measures are simply weak proxies of an underlying uncertainty measure, we stack all of the liquidity and volatility measure to extract what we call the common uncertainty factors.

## 3. CORRELATION ANALYSIS

### 3.1. TIME SERIES PROPERTIES

The autocorrelation function of the first factor for each liquidity and volatility measure, including a two standard deviation band, are plotted in Fig. 1. An AR(2) is fit to each factor series to separate the factors into expected changes and unexpected shocks. Calculating the residuals from the AR(2) regression will yield an estimate of the factor shocks. This is similar to Pástor and Stambaugh (2003) and Archarya and Pedersen (2005) who calculate shocks to liquidity using the residuals to an AR(2) process. The resulting AR(2) estimates are presented in Table 2.

As a measure of the persistence of each factor, the impulse response measured at time $t+12$ to a shock at time $t$ is presented with the AR(2) estimates. The liquidity factors tend to be more persistent than the volatility factors, although both across-measure factors exhibit a degree of persistence. Returns, however, show very little persistence.

### 3.2. CONTEMPORANEOUS CANONICAL CORRELATIONS

The pairwise canonical correlations for the liquidity and volatility factors are calculated using the first three factors for each measure across pairs of measures. This will calculate the maximum correlation between linear combinations of the first three factors for any two measures[3]. The results for the raw factors are presented in Table 3 while Table 4 contains the results for the pre-whitened factors, or factor shocks obtained as the residuals from an AR(2) model.

The correlations for the raw measures are slightly higher than those of the pre-whitened factors. In almost all cases, they are highly correlated especially within the liquidity and volatility groups specifically, although the Amihud measure tends to have a lower correlation with the other measures. The common liquidity and volatility factors are highly correlated with each other as well as the common factor. The individual measures tend to have a higher correlation with their respective "common" factor; for instance the correlation of the Amihud factor with the common (across-measure) liquidity factor is larger than its correlation with the "common" volatility factors. As a whole, Tables 3 and 4 suggest there are strong correlations across all liquidity and volatility factors with most also being highly correlated with the common uncertainty factor. Such

[3]For example, the very first value of Table 3, or 0.098, corresponds to the maximum correlation between a linear combination of the first 3 factors for the Amihud measure and the first three factors of the cross-sectional returns.

large correlations suggest that there is a degree of commonality across the liquidity and volatility measures and that they are contemporaneously correlated with each other and with returns. This result is consistent with recent studies that suggest liquidity and volatility risks are priced factors (see e.g. Pástor and Stambaugh, 2003; Acharya and Pedersen, 2005; Sadka, 2006; Ang et al., 2006; Adrian and Rosenberg, 2006; Moise, 2007).

### 3.3. PREDICTABILITY OF RETURNS, LIQUIDITY, AND VOLATILITY

In the next sections we will examine the relations between liquidity, volatility, and uncertainty on expected returns. Now, we will focus on the relations between shocks to liquidity, volatility, and uncertainty and shocks to future returns. Similarly, we will examine whether shocks to returns affect future shocks to liquidity and volatility. To answer these questions, we examine the pair-wise lead-lag canonical correlations of the shocks to returns and the liquidity, volatility, and uncertainty factors. This is similar to the previous correlation analysis except that one of the factors will be lagged. The results for the raw factors are presented in Table 5 while the lead-lag correlations for the pre-whitened factors (or shocks) are presented in Table 6.

The first column of both Table 5 and Table 6 indicate that there is a weak relation between lagged liquidity and volatility factors returns. The relationship between lagged returns and the liquidity and volatility measures is stronger and suggests that shocks to returns are able to predict shocks to liquidity and volatility. Similarly, lagged liquidity and volatility shocks have a potential for predicting shocks to our aggregate uncertainty factor as seen in the last column of the tables.

Tables 5 and 6 examine the relation strictly focusing on a one month lag. However, predictability might not be restricted to one month but could extend beyond that horizon. The following analysis was performed to determine if predictability might exist beyond a one-month time period. Fig. 2 displays the pairwise canonical lead-lag correlations using the first three factors of each measure. To better understand Fig. 2, let's take a closer look at the plot in the upper left corner. For lag 0, this plot shows the contemporaneous canonical correlation for the shocks of the Amihud factors and the shocks to the returns factors. At lag 4, the plot takes the value of the canonical correlation between the return factors at time $t$ and the Amihud factors at time $t+4$. The slight spike at lag 4 would imply that shocks to returns at time $t$ are correlated with shocks to the Amihud factor four months ahead, at time $t+4$. This suggests that historical returns might be helpful in predicting liquidity. Additionally, when looking at the relationship between volatility measures and returns in Fig. 2 we see that historical volatility shocks might be helpful in predicting returns but there does not appear to be a strong indication that we need to examine periods beyond that of one month when comparing returns with liquidity and volatility. The additional graphs are included for a better understanding of how the shocks to different factors correlate for periods greater than one month but are not the focus of this paper.

The resulting conclusion from the correlations between liquidity and volatility shocks and returns is that liquidity and volatility can be predicted by returns, but the opposite does not appear to hold as the correlations between lagged liquidity and volatility and returns is much weaker. To see this, compare column 1 (the relationship of lagged measures on returns) with row 1 (the correlation of lagged returns on the various measures) of Tables 5 and 6. We now examine the relation of liquidity, volatility, and uncertainty and expected returns.

## 4. JOINT PRICING OF LIQUIDITY AND VOLATILITY RISK IN THE CROSS-SECTION

### 4.1. PORTFOLIO CONSTRUCTION AND TESTING

In this section we examine whether liquidity and volatility risk are jointly priced in the cross section. As has been noted above, several papers (e.g. Ang et al., 2006; Adrian and Rosenberg, 2006; Moise, 2007) have found that volatility risk is priced in the cross-section while others (see Pástor and Stambaugh, 2003; Acharya and Pedersen, 2005; Chen, 2005; Sadka, 2006) have found a similar result for liquidity risk. These papers consider either liquidity or volatility risk separately while Bandi et al. (2008) examines the joint pricing of liquidity and volatility at the market level. They find that when accounting for both risk factors, only volatility is a significant risk factor. They conclude with the thought that liquidity and volatility could both be weak proxies for an underlying uncertainty measure which could explain why only volatility is significant in the joint analysis while both are significant when examined individually.

The first step is to orthogonalize the liquidity and volatility factors from the common uncertainty factor. This will ensure there is no overlap of information contained in the liquidity and volatility shocks with the common uncertainty shocks. Let $\hat{F}_t^{LIQ}$ denote the common liquidity factor with $\hat{F}_t^{VOL}$ as the first across-measure factor of volatility. The first across-measure common uncertainty factor (obtained by taking the first eigenvector of the stacked matrix of liquidity and volatility measures), is denoted as $\hat{F}_t^{U}$. Specifically, the liquidity and volatility factors are orthogonalized using the regression

$$\hat{F}_t^{j} = b_o^j + b_1^j \hat{F}_t^{U} + \hat{u}_t^j \tag{11}$$

where $j = \{LIQ, VOL\}$ and $\hat{u}^j$ is the orthogonalized liquidity (volatility) factor. All of the factors are first pre-whitened using the previous AR(2) specification. By using the pre-whitened factors, we are looking specifically at the factor shocks as opposed to the factors themselves. The following analysis will be focused on the factor shocks and not the raw factors themselves.

The individual liquidity and volatility measures are then regressed on the common uncertainty factor as well as the across-measure liquidity or volatility factor (depending on the group to which it belongs) and the measure specific factor. Both the common liquidity and volatility factors were orthogonalized with the common uncertainty factor. The percentage of firms with significant results, including a test for joint significance, are presented in Table 7. This table represents the relative importance of the different factors in explaining the variation in the firm specific liquidity and volatility measures. As is shown in Table 7, each of the factors is significant at a frequency higher than the test size. Also, for the majority of liquidity and volatility measures, the common uncertainty factor is statistically significant for over 20% of the firms (at the 5% level). The firm-specific volatility measures are impacted by the common uncertainty factor at a higher frequency than the liquidity measures.

To construct our portfolios for the cross-sectional analysis, we first estimate the systematic uncertainty risk using a factor model that includes the three Fama and French (1993) factors (excess market returns (MKT), high-minus-low (HML) book-to-market, and the small-minus-big (SMB) portfolio) and the momentum (UMD) of Carhart (1997).[4] We will collectively refer to these four factors as the FF4 factors. Factor betas are estimated in the first stage regression for each asset through the regression

$$R_{i,t} = \beta_{0,i} + \beta_i' f_t + \varepsilon_{i,t} \tag{12}$$

where $R_{i,t}$ is the excess return of asset $i$ and $f_t$ is a vector of factors. Each month, stocks are ranked according to their uncertainty risk, as measured by their beta on the common uncertainty factor using the previous 36 months. In order for a stock's beta to be estimated in month $t$, we require that there be at least 24 observations within the last 36 months. Based on this beta, the stock is assigned to one of 30 portfolios. Once the portfolios are constructed, the betas for the portfolios, which are assumed to be constant over the sample period, are estimated in the second stage regression using a similar factor model. This means that while the beta of a specific portfolio is assumed to be constant, stocks are allowed to move between portfolios as their specific betas could be changing. In this second stage regression, the orthoganalized liquidity and volatility factors are included in addition to the FF4 and the uncertainty factor.

For each of the 30 portfolios (where stocks are sorted based on exposure to our common uncertainty measure), Table 8 reports the average monthly excess returns, Jensen's $\alpha$ for the factor pricing model using only the FF4 factors, and the post-ranking betas for the liquidity, volatility, and uncertainty factors for each portfolio. The t-statistics are calculated using the standard error adjustment of Newey and West (1987) with 5 lags. The loadings (betas) on the orthogonalized

[4]Thanks to Kenneth French for making these readily available on his website.

liquidity and volatility factors are significant for nearly all of the 30 portfolios while the betas on the uncertainty factor rarely exhibit statistical significance. Additionally, if liquidity, volatility, and uncertainty risk are not priced independently of the FF4 factors, there should be no relation between the portfolio $\alpha$'s and their betas but we find the opposite. Regressing $\alpha$ on the betas yields the following estimates with t-statistics in brackets

$$\alpha_p = \underset{[11.73]}{0.014} - \underset{[-1.31]}{0.013}\beta_{LIQ} + \underset{[8.15]}{0.112}\beta_{VOL} - \underset{[-2.11]}{0.125}\beta_U \qquad R^2 = 0.58. \tag{13}$$

In the next section, we will test explicitly for pricing in the cross-section but the results in Eq. 13 suggest that the common uncertainty factor is significantly priced in the cross-section. Additionally, the volatility specific risk is significant in the cross-section, while liquidity risk might not be significantly priced in the cross-section.

### 4.2. CROSS-SECTIONAL REGRESSIONS

The cross-sectional pricing models testing the pricing of liquidity and volatility risk are of the form

$$\mathrm{E}[R_i] = \lambda_0 + \lambda^{FF'}\beta_i^{FF} + \lambda'\beta_i \tag{14}$$

where $\mathrm{E}[R_i]$ is the expected return of portfolio $i$ in excess of the risk-free rate, $\beta^{FF}$ is the factor loadings for the FF4 factors and $\beta_i$ is the loadings for the liquidity, volatility, and uncertainty risk factors, and $\lambda^{FF}$ and $\lambda$ are vectors of the factor premiums respectively. Specifically, the coefficients are estimated for each month $t = 1, 2, ..., T$ in the cross-sectional specification

$$R_{i,t} = \lambda_{0,t} + \lambda_t^{FF'}\beta_{i,t}^{FF} + \lambda_t'\beta_{i,t} + \nu_{i,t} \tag{15}$$

Eq. (15) is estimated using the method of Fama and MacBeth (1973) where excess returns, $R_{i,t}$ are measured at the firm level. Similar to the previous subsection, firms are sorted into $m$ portfolios based on their exposure to the common uncertainty factor over the previous 36 months. A firm must have at least 24 observations to be included in a portfolio. Once the portfolios are constructed, the betas for each portfolio are assumed to be constant and are calculated over the entire sample. Since there can be much variability in firm specific betas, individual firms are assigned the betas associated with their uncertainty portfolio in month $t$. This procedure is in line with that of Fama and French (1992).

This cross-sectional estimation results in a time series of estimates, $\hat{\lambda}_t^{FF}$ and $\hat{\lambda}_t$. The time-series means and standard errors calculated with a Newey-West correction of 5 lags are presented in Table 9. The results in Table 9 support the conclusion that the significant pricing results can be attributed to the common underlying risk factor and this is consistent across a variety of portfolio sorts. Since the across-measure liquidity and volatility factors are orthogonalized with the common uncertainty factor, we are able to isolate the risk specific to liquidity and volatility. The coefficients on the Common factor for various portfolio sorts are found final column in Table 9. These findings indicate that the combined risk factor from liquidity and volatility measures is significantly priced in the cross-section of returns while the significance of the liquidity and volatility risk is not. None of the loadings on the LIQ or VOL factors are significant while the Common factor has significant loadings in nearly all of the regressions when all risk factors are included regardless of the number of portfolios used. We find the common uncertainty risk factor, extracted across the pooled liquidity and volatility measures, is significantly priced in the cross-section. This implies that the liquidity and volatility could be proxying for an underlying (uncertainty) factor that has significant pricing in the cross-section which would explain the results in Bandi et al. (2008), who find the significance of liquidity risk vanishes once you jointly consider liquidity and volatility. The results are broadly consistent across a range in the number of sorted portfolios and show that the risk specific to liquidity and volatility are not priced, while their common underlying risk (which we have been calling uncertainty) is significantly priced in the cross section.

## 5. CONCLUSION

Several studies find significant systematic liquidity and volatility risk when considered individually. However, liquidity and volatility risk are rarely considered jointly.[5] Bandi et al. (2008) examine liquidity and volatility risk jointly, but only at the market level and over a shorter sample (due to the reliance on high frequency data for their estimation). They find that both liquidity and volatility risk are significant when considered individually, but only volatility risk is significant in the joint specification. Their possible explanation is that liquidity and volatility are both proxies for a significant underlying uncertainty risk of which volatility is a better measure. This paper further examines the relation between liquidity and volatility risk.

We calculate various liquidity and volatility measures across 4975 NYSE firms from July 1962 to December 2011. Latent factor models are estimated for each measure. Additionally, the latent factors of the pooled liquidity (volatility) measures are extracted to form across-measure liquidity (volatility) factors. To explore the possibility that they are both proxies for an underlying uncertainty factor, a latent factor model is estimated across the collection of both liquidity and volatility measures. We find that there is a high correlation between the common uncertainty factor and the individual liquidity and volatility measures. Shocks to returns are contemporaneously correlated to shocks to individual liquidity and volatility measures as well as shocks to the across-measure liquidity and volatility factors. Additionally, there is evidence that shocks to returns can predict shocks to both liquidity and volatility across assets. Liquidity shocks are very persistent while shocks to volatility tend to have no impact after about 12 months.

For the cross-sectional pricing analysis, the across-measure liquidity and volatility risk factors are orthogonalized from the uncertainty risk factor. Neither the liquidity specific risk factor nor the volatility specific risk factor exhibit significant pricing in the cross-section, while the common uncertainty risk is significant in the cross-sectional specification. These results indicate that both

[5] Amihud (2002) does control for the raw annual volatility in his analysis of liquidity risk.

liquidity and volatility are proxies for an underlying and significant risk factor, which we term "uncertainty." Furthermore, the significant results in the liquidity and volatility literatures appear to result from the ability of the various liquidity and volatility measures to proxy for the underlying uncertainty risk.

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# APPENDIX

Table 1: Degree of commonality in the measure specific factors.
This table reports the average $R^2$ for each stock's time series regression of its measure on the corresponding factors. A total of 3 factors were extracted for four liquidity, three volatility estimators, and the monthly returns using the method of Connor and Korajczyk (1987) for unbalanced panels. Each measure was normalized by its cross-sectional mean and standard deviation at time $t-1$. The total sample included 4975 stocks from the NYSE spanning July 1962 to December 2011.

| Measure | 1 Factor | 2 Factors | 3 Factors |
|---|---|---|---|
| Amihud | 0.4267 | 0.5071 | 0.5912 |
| RS | 0.3819 | 0.5379 | 0.5917 |
| Roll | 0.1614 | 0.3303 | 0.3583 |
| Turnover | 0.3523 | 0.4624 | 0.4956 |
| RV | 0.2918 | 0.4289 | 0.4902 |
| GK | 0.3207 | 0.4714 | 0.5269 |
| Garch | 0.4857 | 0.6291 | 0.6993 |
| Returns | 0.2423 | 0.2621 | 0.2924 |

Table 2: AR(2) Results.
AR(2) estimates for the first factor of each liquidity and volatility measure with the corresponding t-stat in parentheses. The common liquidity, volatility, and uncertainty factors are also estimated. The impulse response measure the fraction of a time $t$ shock that remains after 12 periods (one year). The liquidity factors exhibit higher persistence than the volatility factors.

| Measure | $\phi_1$ | $\phi_2$ | Impulse Response | Measure | $\phi_1$ | $\phi_2$ | Impulse Response |
|---|---|---|---|---|---|---|---|
| Amihud | 0.5641 | 0.4115 | 0.5695 | RV | 0.6181 | 0.1597 | 0.0685 |
| | (31.20) | (24.64) | | | (65.51) | (8.95) | |
| RS | 0.7700 | 0.1470 | 0.3504 | GK | 0.7556 | 0.1555 | 0.0850 |
| | (41.23) | (7.10) | | | (131.38) | (20.49) | |
| Roll | 0.5732 | 0.4060 | 0.5901 | Garch | 0.6383 | 0.3292 | 0.3270 |
| | (25.45) | (18.12) | | | (51.57) | (24.03) | |
| Turnover | 0.5140 | 0.4718 | 0.6011 | VOL | 0.6643 | 0.1340 | 0.2580 |
| | (28.52) | (24.99) | | | (78.39) | (10.18) | |
| LIQ | 0.1949 | -0.0541 | 0.6112 | Returns | 0.6650 | 0.1334 | 0.0000 |
| | (5.23) | (-1.54) | | | (77.96) | (10.10) | |
| Common | 0.6794 | 0.2100 | 0.5531 | | | | |
| | (68.30) | (16.48) | | | | | |

Table 3: Pairwise Contemporaneous Canonical Correlations (Raw Series).
Three common factors are extracted from a variety of liquidity and volatility measures. The liquidity measures are the Amihud (2002) measure, the relative spread, the Roll (1984) measure, and the turnover. Monthly volatility estimates include monthly realized variance (RV), the monthly Garman-Klass estimate, and the estimates from a GARCH(1,1) model. The sample includes 4975 NYSE stocks from July 1962 to December 2011. The contemporaneous, pairwise canonical correlation for the first three raw factors of each measure is presented below.

| | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL |
|---|---|---|---|---|---|---|---|---|---|---|
| Amihud | 0.098 | | | | | | | | | |
| RS | 0.213 | 0.802 | | | | | | | | |
| Roll | 0.199 | 0.876 | 0.952 | | | | | | | |
| Turnover | 0.292 | 0.693 | 0.881 | 0.727 | | | | | | |
| RV | 0.291 | 0.741 | 0.931 | 0.891 | 0.685 | | | | | |
| GK | 0.294 | 0.745 | 0.962 | 0.861 | 0.795 | 0.972 | | | | |
| Garch | 0.218 | 0.868 | 0.867 | 0.791 | 0.777 | 0.748 | 0.773 | | | |
| LIQ | 0.139 | 0.973 | 0.929 | 0.936 | 0.985 | 0.852 | 0.865 | 0.862 | | |
| VOL | 0.408 | 0.663 | 0.955 | 0.894 | 0.765 | 0.990 | 0.995 | 0.990 | 0.846 | |
| Common | 0.230 | 0.881 | 0.977 | 0.921 | 0.958 | 0.966 | 0.963 | 0.902 | 0.984 | 0.983 |

Table 4: Pairwise Contemporaneous Canonical Correlations Pre-whitened Factors (shocks)
This table contains the lead-lag canonical correlations for the shocks to the liquidity and volatility factors as estimated using an AR(2) model. Please see Table 5 for a detailed description of the raw factors.

| | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL |
|---|---|---|---|---|---|---|---|---|---|---|
| Amihud | 0.149 | | | | | | | | | |
| RS | 0.365 | 0.121 | | | | | | | | |
| Roll | 0.322 | 0.257 | 0.800 | | | | | | | |
| Turnover | 0.349 | 0.284 | 0.684 | 0.464 | | | | | | |
| RV | 0.294 | 0.111 | 0.875 | 0.756 | 0.544 | | | | | |
| GK | 0.305 | 0.093 | 0.890 | 0.727 | 0.599 | 0.946 | | | | |
| Garch | 0.130 | 0.918 | 0.103 | 0.120 | 0.168 | 0.234 | 0.110 | | | |
| LIQ | 0.261 | 0.918 | 0.910 | 0.719 | 0.959 | 0.774 | 0.800 | 0.085 | | |
| VOL | 0.311 | 0.060 | 0.885 | 0.729 | 0.607 | 0.973 | 0.983 | 0.939 | 0.805 | |
| Common | 0.322 | 0.511 | 0.925 | 0.732 | 0.875 | 0.952 | 0.959 | 0.264 | 0.952 | 0.986 |

Table 5: Lead-lag Canonical Correlations Raw Factors.
Three common factors are extracted separately for multiple liquidity and volatility measures in addition to returns. Across-measure liquidity (volatility) factors were estimated for the combined liquidity (volatility) measures. Common "uncertainty" factors were extracted across all liquidity and volatility measures. The measures were standardized by their cross-sectional means and standard deviations before the factor analysis to eliminate differing units of measure. The liquidity factors considered are the Amihid (2002) measure (daily absolute return divided by the dollar volume), the relative spread (bid-ask spread divided by its mean), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of monthly volume and shares outstanding). The volatility measures are monthly realized variance (sum of daily squared returns), the Garman-Klass measure (based on the daily high, low, open, and close), and the estimates of conditional variance of a GARCH(1,1) model. The sample includes 4975 MYSE firms over the period of July 1962 to December 2011.

| $t-1\backslash t$ | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Return | 0.22 | 0.20 | 0.27 | 0.26 | 0.32 | 0.29 | 0.28 | 0.34 | 0.22 | 0.47 | 0.25 |
| Amihud | 0.09 | 0.95 | 0.81 | 0.87 | 0.69 | 0.73 | 0.74 | 0.87 | 0.91 | 0.66 | 0.84 |
| RS | 0.14 | 0.80 | 0.97 | 0.86 | 0.87 | 0.85 | 0.89 | 0.90 | 0.91 | 0.88 | 0.94 |
| Roll | 0.13 | 0.87 | 0.87 | 0.92 | 0.69 | 0.81 | 0.79 | 0.82 | 0.90 | 0.80 | 0.87 |
| Turnover | 0.15 | 0.69 | 0.87 | 0.72 | 0.97 | 0.69 | 0.77 | 0.78 | 0.94 | 0.76 | 0.93 |
| RV | 0.15 | 0.71 | 0.85 | 0.81 | 0.64 | 0.87 | 0.78 | 0.83 | 0.80 | 0.86 | 0.86 |
| GK | 0.14 | 0.77 | 0.90 | 0.82 | 0.77 | 0.89 | 0.88 | 0.81 | 0.85 | 0.87 | 0.85 |
| Garch | 0.20 | 0.86 | 0.84 | 0.77 | 0.78 | 0.71 | 0.75 | 0.98 | 0.85 | 0.78 | 0.88 |
| LIQ | 0.09 | 0.91 | 0.91 | 0.89 | 0.94 | 0.82 | 0.84 | 0.88 | 0.96 | 0.83 | 0.96 |
| VOL | 0.24 | 0.67 | 0.89 | 0.80 | 0.72 | 0.88 | 0.82 | 0.87 | 0.82 | 0.91 | 0.90 |
| Common | 0.12 | 0.84 | 0.94 | 0.87 | 0.92 | 0.85 | 0.85 | 0.92 | 0.96 | 0.90 | 0.97 |

Table 6: Lead-lag Canonical Correlations Pre-whitened Factors (shocks).
This table contains the lead-lag canonical correlations for the shocks to the liquidity and volatility factors as estimated using an AR(2) model. Please see Table 5 for a detailed description of the raw factors.

| $t-1\backslash t$ | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Return | 0.16 | 0.39 | 0.23 | 0.32 | 0.20 | 0.22 | 0.25 | 0.34 | 0.40 | 0.32 | 0.24 |
| Amihud | 0.20 | 0.21 | 0.13 | 0.14 | 0.22 | 0.13 | 0.08 | 0.23 | 0.30 | 0.06 | 0.28 |
| RS | 0.14 | 0.20 | 0.16 | 0.12 | 0.24 | 0.26 | 0.19 | 0.36 | 0.19 | 0.20 | 0.24 |
| Roll | 0.18 | 0.23 | 0.10 | 0.13 | 0.19 | 0.23 | 0.11 | 0.31 | 0.14 | 0.18 | 0.21 |
| Turnover | 0.15 | 0.15 | 0.15 | 0.15 | 0.25 | 0.22 | 0.18 | 0.29 | 0.23 | 0.18 | 0.20 |
| RV | 0.11 | 0.17 | 0.17 | 0.16 | 0.27 | 0.41 | 0.20 | 0.48 | 0.15 | 0.26 | 0.22 |
| GK | 0.16 | 0.27 | 0.19 | 0.21 | 0.26 | 0.39 | 0.29 | 0.34 | 0.24 | 0.29 | 0.31 |
| Garch | 0.13 | 0.15 | 0.16 | 0.16 | 0.13 | 0.17 | 0.11 | 0.22 | 0.16 | 0.21 | 0.21 |
| LIQ | 0.19 | 0.26 | 0.15 | 0.18 | 0.24 | 0.21 | 0.17 | 0.37 | 0.32 | 0.16 | 0.31 |
| VOL | 0.18 | 0.16 | 0.21 | 0.20 | 0.25 | 0.36 | 0.19 | 0.39 | 0.18 | 0.26 | 0.29 |
| Common | 0.18 | 0.26 | 0.19 | 0.16 | 0.25 | 0.33 | 0.22 | 0.41 | 0.26 | 0.26 | 0.27 |

Table 7: Percent of firms with significant exposure to the common uncertainty factor.
Each measure is regressed on the common uncertainty factor, the across-measure liquidity or volatility factor, and its own measure-specific factor. Each factor is pre-whitened using an AR(2) specification. The measure-specific and liquidity/volatility factors were orthogonalized to the common uncertainty factor using a specification similar to Eq. (11).The table reports the percentage of firms with a significant coefficient at the 5% level. It also includes the percentage of firms where the test of joint significance exceeds the 5% level. The average $R^2$ is also included. The sample contains 4975 NYSE firms over the period from July 1962 to December 2011.

| Variable | LIQ/VOL measure | Common measure | Specific measure | Joint Sign. | Average $R^2$ |
|---|---|---|---|---|---|
| Amihud | 3.0 | 5.2 | 20.1 | 14.5 | 0.05 |
| RS | 9.8 | 42.3 | 42.7 | 61.2 | 0.11 |
| Roll | 11.5 | 17.0 | 9.3 | 26.5 | 0.08 |
| Turnover | 8.3 | 33.4 | 22.7 | 35.4 | 0.06 |
| RV | 16.7 | 56.4 | 46.6 | 73.5 | 0.22 |
| GK | 23.2 | 59.6 | 18.1 | 63.4 | 0.19 |
| GARCH | 51.2 | 22.1 | 23.4 | 49.9 | 0.08 |

Table 8: Portfolios formed by sorting on the common uncertainty factor.
Across-measure common factors (which we refer to as uncertainty factors) are jointly extracted for various liquidity and volatility measures. Each stock is then assigned to one of 30 portfolios based on its exposure to this common factor over the previous 36 months (a minimum of 24 observations is required). The excess returns for these 20 portfolios are then regressed on the FF4 factors (MKT, HML, SMB, and UMD) and the liquidity, volatility, and uncertainty factors. The liquidity measures considered are the Amihud (2002) measure, defined as the absolute return divided by dollar volume, the relative spread, the Roll (1984) measure, and turnover. The volatility measures are realized variance, the Garman and Klass (1980) estimate, and the conditional volatility estimated from a simple GARCH(1,1) model. Before extracting a common "uncertainty" factor across all of these measures, they are each standardized by their respective cross-sectional means and standard deviations. The sample consists of 4975 NYSE stocks from July 1962 to December 2011.

| Portfolio Ranking | Excess Return | t-stat | FF4 $\alpha$ | t-stat | $\beta_{LIQ}$ | t-stat | $\beta_{VOL}$ | t-stat | $\beta_{ALL}$ | t-stat |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0195 | 4.5046 | 0.0194 | 3.9793 | 0.3005 | 2.5683 | 0.1197 | 1.6427 | 0.0407 | 1.3721 |
| 2 | 0.0147 | 4.4043 | 0.0144 | 3.8573 | 0.0607 | 0.6934 | -0.0186 | -0.4086 | 0.0187 | 0.8644 |
| 3 | 0.0118 | 4.1024 | 0.0103 | 3.3424 | -0.0120 | -0.1517 | -0.0452 | -1.0335 | 0.0085 | 0.5515 |
| 4 | 0.0111 | 4.2796 | 0.0099 | 3.6586 | -0.0289 | -0.4320 | -0.0322 | -0.7852 | 0.0118 | 1.0319 |
| 5 | 0.0064 | 2.5426 | 0.0056 | 2.0438 | -0.0515 | -0.8627 | -0.0471 | -1.3493 | 0.0149 | 1.0463 |
| 6 | 0.0097 | 3.8909 | 0.0085 | 2.9485 | -0.0571 | -0.8448 | -0.0558 | -1.3951 | 0.0117 | 0.8073 |
| 7 | 0.0089 | 3.6661 | 0.0079 | 2.9390 | -0.0582 | -1.0794 | -0.0403 | -1.2108 | 0.0105 | 0.9081 |
| 8 | 0.0065 | 2.9962 | 0.0060 | 2.3881 | -0.0937 | -1.8569 | -0.0555 | -1.7720 | 0.0121 | 0.9311 |
| 9 | 0.0089 | 4.1059 | 0.0077 | 3.0825 | -0.1023 | -1.8623 | -0.0613 | -1.8109 | 0.0110 | 1.0282 |
| 10 | 0.0080 | 3.7476 | 0.0079 | 3.2045 | -0.0951 | -1.7182 | -0.0584 | -1.7789 | 0.0101 | 0.9057 |
| 11 | 0.0079 | 3.8086 | 0.0071 | 3.0728 | -0.0715 | -1.3453 | -0.0465 | -1.4105 | 0.0115 | 1.1550 |
| 12 | 0.0080 | 3.8019 | 0.0074 | 3.1624 | -0.1392 | -2.8585 | -0.0590 | -2.0424 | 0.0127 | 1.3342 |
| 13 | 0.0075 | 3.6358 | 0.0074 | 3.2140 | -0.1157 | -2.0838 | -0.0655 | -2.0024 | 0.0133 | 1.5208 |
| 14 | 0.0073 | 3.5868 | 0.0067 | 3.1835 | -0.0913 | -2.1041 | -0.0376 | -1.4407 | 0.0105 | 1.3345 |
| 15 | 0.0078 | 3.8780 | 0.0070 | 3.2427 | -0.1171 | -2.4847 | -0.0551 | -1.8563 | 0.0075 | 0.9405 |
| 16 | 0.0068 | 3.4204 | 0.0060 | 2.7278 | -0.0943 | -2.1498 | -0.0385 | -1.5320 | 0.0170 | 2.0419 |
| 17 | 0.0098 | 4.7877 | 0.0094 | 4.1378 | -0.1477 | -3.2179 | -0.0572 | -2.1802 | 0.0049 | 0.5834 |
| 18 | 0.0084 | 4.0529 | 0.0074 | 3.2429 | -0.1019 | -2.2914 | -0.0418 | -1.5501 | 0.0105 | 1.1511 |
| 19 | 0.0085 | 4.1794 | 0.0083 | 3.8859 | -0.1434 | -3.3938 | -0.0580 | -2.3862 | 0.0166 | 2.3591 |
| 20 | 0.0091 | 4.5246 | 0.0081 | 3.8605 | -0.1403 | -3.4548 | -0.0458 | -1.8923 | 0.0096 | 1.2387 |
| 21 | 0.0089 | 4.2084 | 0.0083 | 3.4783 | -0.1248 | -3.1453 | -0.0439 | -1.7829 | 0.0112 | 1.2800 |
| 22 | 0.0086 | 4.1657 | 0.0078 | 3.3650 | -0.1514 | -3.7293 | -0.0553 | -2.5299 | 0.0102 | 1.1763 |
| 23 | 0.0081 | 3.7887 | 0.0073 | 3.1654 | -0.1604 | -3.9728 | -0.0515 | -2.1747 | 0.0091 | 1.0759 |
| 24 | 0.0098 | 4.5008 | 0.0087 | 3.7648 | -0.1666 | -3.2526 | -0.0574 | -1.9052 | 0.0117 | 1.3508 |
| 25 | 0.0117 | 4.9659 | 0.0106 | 4.1432 | -0.1664 | -3.1707 | -0.0543 | -1.7988 | 0.0053 | 0.5836 |
| 26 | 0.0108 | 4.4781 | 0.0088 | 3.1468 | -0.2020 | -3.3677 | -0.0786 | -2.5314 | 0.0090 | 0.7700 |
| 27 | 0.0119 | 4.6788 | 0.0105 | 3.7298 | -0.1591 | -3.0534 | -0.0427 | -1.5182 | 0.0162 | 1.4848 |
| 28 | 0.0137 | 4.9547 | 0.0126 | 4.1850 | -0.1903 | -2.8202 | -0.0494 | -1.3322 | 0.0123 | 1.0475 |
| 29 | 0.0166 | 5.1151 | 0.0147 | 4.1660 | -0.2577 | -3.1010 | -0.0778 | -1.6780 | 0.0023 | 0.1551 |
| 30 | 0.0197 | 4.7515 | 0.0174 | 4.1643 | -0.2243 | -2.0289 | -0.0097 | -0.1484 | -0.0094 | -0.5549 |

Table 9: Pricing uncertainty in the cross-section.
Factors are extracted for multiple liquidity and volatility measures. Additionally, common liquidity (volatility) factors across all liquidity (volatility) measures are extracted. Finally, uncertainty factors are obtained from the collection of liquidity and volatility measures. Each firm is sorted into one of $m$ portfolios based on its individual exposure to the common uncertainty factor (estimated using a rolling 36 month window in which firms are required to have a minimum of 24 months of observations). The results of cross-sectional regressions of excess returns on the factor loadings (betas) is presented below. Since the loadings at the firm level are nosier than the loadings at the portfolio level, each firm is assigned the vector of betas of its portfolio in month $t$. Before performing the factor analysis, each measure is standardized by its cross-sectional means standard deviations. The liquidity measures include the Amihud (2002) measure (sum of absolute returns divided by dollar volume), the relative spread (the ratio of bid-ask spread and the average of the bid and ask), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of volume and shares outstanding). The volatility measures are realized variance (sum of squared daily returns), the Garman and Klass (1980) measure (based on the high, low, open, and close price), and the conditional volatility estimates of a monthly GARCH(1,1) model. The sample consists of 4975 MYSE stocks from July 1962 to December 2011. T-statistics using a Newey-West correction of 5 lags are included in brackets.

| | MKT | HML | SMB | MOM | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|
| $m = 70$ | -0.4275 | | | | -0.0041 | 0.0212 | -0.0392 |
| | [-1.13] | | | | [-0.80] | [1.51] | [-1.63] |
| | -0.2337 | 0.2702 | -0.0037 | 0.5179 | -0.0033 | 0.0191 | -0.0518 |
| | [-0.56] | [0.99] | [-0.02] | [1.39] | [-0.66] | [1.59] | [-2.00] |
| $m = 50$ | -0.6805 | | | | -0.0032 | 0.0123 | -0.0363 |
| | [-1.24] | | | | [-0.65] | [0.94] | [-1.12] |
| | -0.2469 | 0.1129 | 0.1719 | 0.8210 | 0.0005 | 0.0089 | -0.0747 |
| | [-0.51] | [0.43] | [0.72] | [1.86] | [0.10] | [0.68] | [-1.80] |
| $m = 25$ | -1.4810 | | | | -0.0036 | 0.0046 | -0.0588 |
| | [-1.92] | | | | [-0.65] | [ 0.25] | [-1.38] |
| | 0.1478 | 0.8664 | -0.0908 | 1.3348 | -0.0002 | 0.0126 | -0.1486 |
| | [0.21] | [ 2.56] | [-0.32] | [1.74] | [-0.03] | [0.71] | [-2.28] |
| $m = 20$ | -1.3987 | | | | 0.0004 | -0.0079 | -0.0748 |
| | [-1.68] | | | | [0.07] | [-0.38] | [-1.72] |
| | -0.4349 | 0.3862 | 0.2551 | 0.5268 | 0.0020 | -0.0007 | -0.0779 |
| | [-0.61] | [1.02] | [0.64] | [0.89] | [0.33] | [-0.03] | [-1.57] |
| $m = 15$ | -1.6116 | | | | -0.0032 | 0.0049 | -0.0489 |
| | [-2.09] | | | | [-0.44] | [0.18] | [-1.48] |
| | -0.2018 | 0.0770 | 0.0822 | 1.2788 | 0.0033 | 0.0047 | -0.0904 |
| | [-0.23] | [0.15] | [0.23] | [1.40] | [0.43] | [0.14] | [-1.94] |

Figure 1: Autocorrelations of liquidity and volatility factors.
Common factors are extracted for liquidity measures, volatility measures, and returns. The autocorrelations of the first factor for each measure are depicted in the below plots. The liquidity measures are the Amihud measure, relative spread (RS), the Roll measure, and turnover. The measures of volatility include monthly realized variance (RV), the Garman-Klass (GK) estimate, and monthly GARCH(1,1) estimates. The sample includes 4975 NYSE stocks from July 1962 to December 2011.

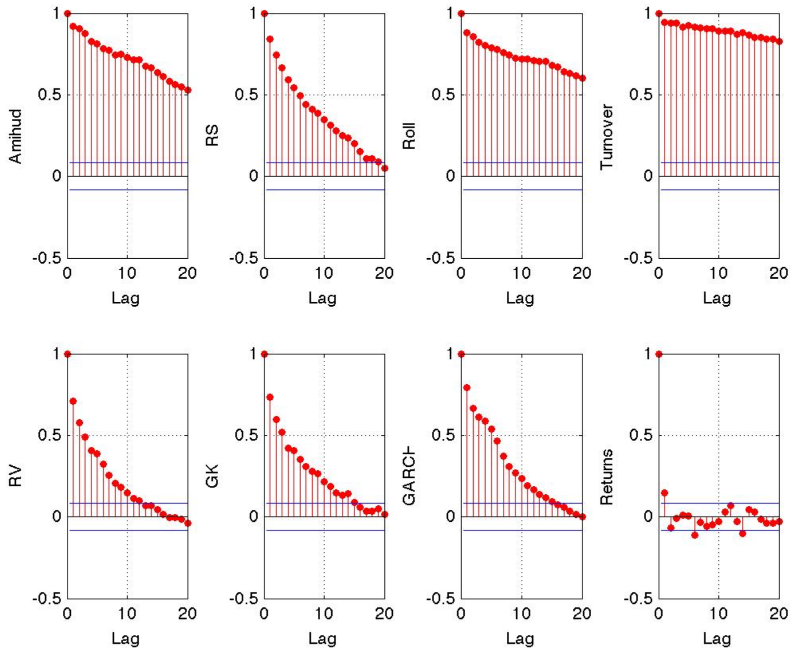

Figure 2: Canonical lead-lag correlations using the first three factors.
Common factors are extracted for liquidity measures, volatility measures, and returns. Pairwise lead-lag canonical correlations (for 15 leads and lags) of the first 3 factors for each measure are plotted below. Each measure is standardized by its mean and standard deviation before performing the factor analysis. Additionally, the factors are pre-whitened using an AR(2) specification. The liquidity measures are the Amihud measure, relative spread (RS), the Roll measure, and turnover. The measures of volatility include monthly realized variance (RV), the Garman-Klass (GK) estimate, and monthly GARCH(1,1) estimates. The sample includes 4975 NYSE stocks from July 1962 to December 2011.